An Exact TEM Calculation of Loss in a Stripline of Arbitrary Dimensions

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Abstract—**An exact expression for the quasi-static conductive attenuation in a symmetrical stripline is derived. The formulation is based on a TEM assumption, which assumes that the skin depth is much smaller than the strip thickness. The conductive attenuation is related to the charge density on the conductive surfaces, which is determined by a conformal mapping originally proposed by Bates. An analytic extraction of a charge singularity term is used to obtain a numerically efficient calculation, in which no singular integrations appear.**

I. Introduction

Symmetrical stripline, shown in Fig. 1, is one of the most common transmission lines in use at microwave frequencies. The dominant mode on this structure is approximately TEM, provided the conductor losses are small. Because of this, a quasi-static analysis may be used to determine both the characteristic impedance Z_0 and the attenuation constant α [1]. An exact calculation of Z_0 using a conformal mapping method was originally given by Bates [2]. The total attenuation constant α is in general the sum of a conductive attenuation constant α_c and a dielectric attenuation constant α_d, with α_d given (for low loss) by the simple expression [1]

$$\alpha_d = k_0 \frac{\sqrt{\epsilon_r}}{2} \tan \delta_d \tag{1}$$

where $\tan \delta_d$ is the loss tangent of the stripline medium and k_0 is the free-space wavenumber.

For the conductive attenuation, the Wheeler incremental inductance rule [3] was applied by Cohn [4] to obtain approximate formulas for α_c. In particular, two formulas were obtained; a narrow strip formula for $w/b \leqslant 0.35$ and a wide strip formula for $w/b \geqslant 0.35$. Although the Wheeler incremental inductance rule is exact, the Cohn formulas are approximate since approximate formulas for the line characteristic impedance were used in the calculation. In principle, an exact calculation of α_c could be obtained by using an exact formula for Z_0. However, the Z_0 formula derived in [2] is not in the form of an explicit function of the stripline dimensions, which would make such a calculation extremely difficult.

In the present work, an alternative approach is adopted, in which the conductive attenuation constant is calculated from

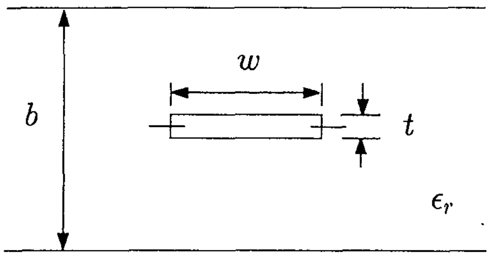

Fig. 1. Geometry of a symmetrical stripline.

the well-known quasi-static formula

$$\alpha_c = \frac{R_s}{2Z_0} \int_{\Gamma_T} \frac{\rho_s^2 d\ell}{Q^2} \tag{2}$$

where ρ_s is the surface charge density on all parts of the conductive system, Q is the total charge on the center conductor, and R_s is the surface resistance of the metal. The total contour Γ_T includes both the center conductor and the metal ground planes.

In order to determine the charge density ρ_s, the conformal mapping of Bates is used [2]. In this transformation, the upper half of the original stripline is mapped into a coaxial cylinder geometry, in which the charge density is uniform. The integration in (2) is then transformed into an integration along the circular boundaries of the coaxial geometry. In this integration, the integrand has an integrable singularity which corresponds to the charge singularity at the corners of the stripline center conductor. In order to obtain an efficient calculation, this singularity is extracted from the integral and evaluated analytically.

Results are presented for several cases, and compared with the Cohn formulas. It is concluded that the maximum error in the Cohn formulas occurs at the joining part $w/b = 0.35$, and is typically 5–6%. In addition to providing accurate values of α_c for design purposes, results from the present method may also be used to benchmark results from other methods in the future.

II. Conformal Mapping of Stripline

To determine ρ_s appearing in (2), a series of conformal mappings originally used by Bates [2] is applied, as shown in Fig. 2. The upper half of the stripline in the z-plane is first mapped into the upper half of the p-plane through the

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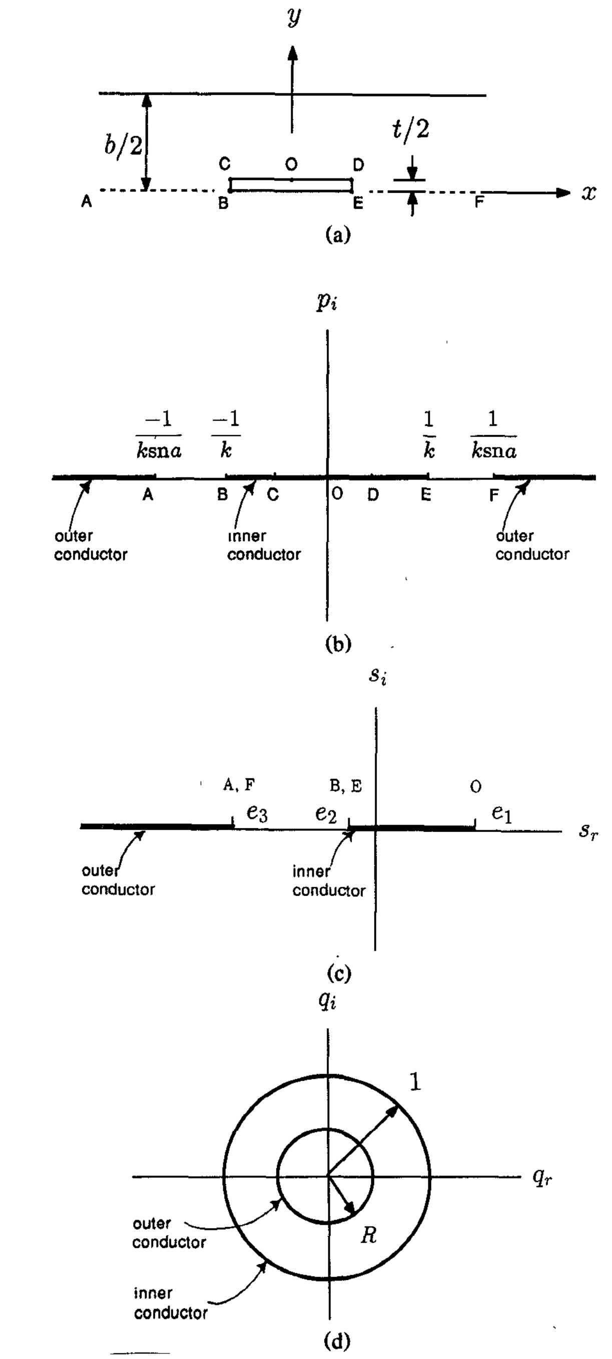

Fig. 2. The conformal mappings used to transform the upper half of the original stripline into a pair of coaxial cylinders. The mappings of the inner conductor (strip) and outer conductor (top ground plane) are indicated in each plane. (a) The z plane. (b) The p plane. (c) The s plane. (d) The q plane.

transformation

$$\frac{dz}{dp}=\frac{(1-p^2)^{1/2}}{(1-k^2p^2)^{1/2}(1-k^2p^2\,\mathrm{sn}^2(a))} \tag{3}$$

where $\mathrm{sn}(a)=\mathrm{sn}(a,k)$ is the Jacobian elliptic function of the first kind with argument a and parameter k. The constants a and k will eventually be related to Z_0 and w/b, but for now are left as arbitrary parameters.

The upper half of the p-plane is then mapped into the complete s-plane through the transformation

$$s=-p^2C_1+C_0 \tag{4}$$

where C_0 and C_1 are left unspecified for the moment. Equation (4) is a slight generalization of the transformation $s=p^2$ originally used by Bates. This generalization is not important for the calculation of Z_0, but is required in the calculation of α_c where absolute dimensions are required. The values of s at the conductor edges in the s-plane are denoted e_1, e_2, e_3 and are given as

$$e_1=C_0 \tag{5}$$

$$e_2=C_0-\frac{C_1}{k^2} \tag{6}$$

$$e_3=C_0-\frac{C_1}{k^2\,\mathrm{sn}^2(a)}. \tag{7}$$

Finally, the s-plane is mapped into the q-plane with the transformation [5, p. 191]

$$s=P\left(\ln\left(\frac{q}{R}\right)\right) \tag{8}$$

where $P(z)=P(z;e_1,e_2,e_3)$ is the Weierstrass elliptic function with parameters e_1,e_2,e_3 [6, pp. 307–314]. An efficient calculation of this function is discussed in the Appendix. The Weierstrass elliptic function requires in its definition that

$$e_1+e_2+e_3=0. \tag{9}$$

Enforcing this condition with Eqs. (5)–(7) yields

$$C_0=\frac{1}{3}\left(\frac{1}{k^2}+\frac{1}{k^2\,\mathrm{sn}^2(a)}\right)C_1. \tag{10}$$

The constant C_0 is thus related to C_1 which is still arbitrary (but which will be related to Z_0 presently). Another set of conditions, imposed on the Weierstrass function for the mapping of Fig. 2(d) to be valid, is [5, p. 191]

$$w_1=\ln\left(\frac{1}{R}\right) \tag{11}$$

$$w_2=j\pi \tag{12}$$

where w_1 and w_2 are the real and imaginary half-periods of the Weierstrass function, given as [6, p. 308]

$$w_1=\frac{K(k_2)}{\sqrt{e_1-e_3}} \tag{13}$$

$$w_2=\frac{jK'(k_2)}{\sqrt{e_1-e_3}} \tag{14}$$

where

$$k_2=\sqrt{\frac{e_2-e_3}{e_1-e_3}} \tag{15}$$

and $K(k_2)$ is the complete elliptic integral of the first kind (see the Appendix) and

$$K'(k_2)=K(k_2')=K\left(\sqrt{1-k^2}\right). \tag{16}$$

Enforcing (11), and using (13) together with (5)–(7) results in

$$C_1=k^2r^2 \tag{17}$$

where

$$r = \frac{K(k_2)\,\mathrm{sn}(a)}{\ln\left(\dfrac{1}{R}\right)}. \tag{18}$$

Dividing (11) by (12), and using (13), (14) yields

$$\frac{K(k_2)}{K'(k_2)} = \frac{1}{\pi}\ln\left(\frac{1}{R}\right) = \frac{4Z_0}{\zeta} \tag{19}$$

where

$$Z_0 = \frac{1}{2}Z_0^q = \frac{\zeta}{4\pi}\ln\left(\frac{1}{R}\right) \tag{20}$$

and

$$\zeta = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}}. \tag{21}$$

The term Z_0^q is the characteristic impedance of the coaxial line (which gets mapped into half the stripline). Substituting (5)–(7) into (15) and simplifying,

$$k_2^2 = 1 - \mathrm{sn}^2(a) = \mathrm{cn}^2(a,k). \tag{22}$$

Equations (19) and (22) imply that the parameter $k_2 = \mathrm{cn}(a,k)$ is directly related to Z_0. The independent parameter k is related to the aspect ratio w/b (different aspect ratios may give the same Z_0). Using the geometry of Fig. 2(a), the transformations of Fig. 2 relate the strip dimensions to k and a as [2].

$$\frac{w}{b} = \frac{2K(k)}{\pi}\left[\frac{k^2\,\mathrm{sn}(a)\,\mathrm{cn}(a)}{\mathrm{dn}(a)} - Z(a)\right] \tag{23}$$

$$\frac{t}{b} = \frac{a}{K(k)} - \frac{2K'}{\pi}\left[\frac{k^2\,\mathrm{sn}(a)\,\mathrm{cn}(a)}{\mathrm{dn}(a)} - Z(a)\right] \tag{24}$$

$$b = \frac{\pi\,\mathrm{dn}(a)}{k^2\,\mathrm{sn}(a)\,\mathrm{cn}(a)} \tag{25}$$

where $Z(a)$ is the Jacobian Zeta function (see the Appendix) and

$$\mathrm{dn}(a) = \sqrt{1 - k^2\,\mathrm{sn}^2(a,k)}. \tag{26}$$

For a given Z_0, k determines both w/b and t/b. The actual value of b, given in (25), is not important for impedance calculations, but is for attenuation.

III. Calculation of Attenuation

Assuming one Coulomb/meter on each conductor in the q plane, the charge density may be expressed as

$$\rho_s^q = \frac{1}{2\pi|q|} \tag{27}$$

where $|q| = 1$ for the outer coax, and $|q| = R$ for the inner coax. In the z plane,

$$\rho_s^z = \rho_s^q\left|\frac{dq}{dz}\right| = \rho_s^q\left|\frac{dq}{ds}\frac{ds}{dp}\frac{dp}{dz}\right|. \tag{28}$$

The last derivative is given by (3), while the second one is found directly from (4). To compute the first one, note from (8) that

$$\frac{ds}{dq} = \frac{1}{q}P'(t) \tag{29}$$

where $t = \ln(q/R)$. The derivative of the Weierstrass function is (see the Appendix)

$$\frac{ds}{dt} = P'(t) = 2\sqrt{(s-e_1)(s-e_2)(s-e_3)} \tag{30}$$

where $s = P(t)$. Using this in (28) and using (4), (17) allows ρ_s^z to be expressed, after simplification, as

$$\rho_s^z = \frac{1}{2\pi}\frac{k^2\,\mathrm{sn}(a)}{r}\left(\frac{|r^2 + \mathrm{sn}^2(a)(s-e_1)|}{|k^2r^2 + (s-e_1)|}\right). \tag{31}$$

Equation (31) is valid on both inner strip conductor (outer coax) and the ground planes (inner coax), because the q term in (29), when used in (28), cancels the $|q|$ term which appears in (27). The value of s is different on the two conductors, however.

The charge density from (31) could be used in (2) to compute the attenuation, but it is not convenient to carry out the integration over the infinite ground planes. Equation (2) may be transformed into integrations over the coaxial cylinders of Fig. 2(d) however, corresponding to the top ground plane and half the strip. Denote $C = C_1$ or C_2 as either the outer or inner coaxial cylinder, corresponding to the contour $\Gamma = \Gamma_1$ or Γ_2 in the z-plane, which is either the half-strip or the top ground plane shown in Fig. 1(a). The required integral term (called I) may then be expressed as

$$\begin{aligned} I &\cong \int_\Gamma |\rho_s^z|^2 |dz| \\ &= \int_\Gamma\left[\frac{1}{2\pi|q|}\left|\frac{dq}{dz}\right|\right]^2 |dz| \\ &= \int_C \frac{1}{(2\pi|q|)^2}\left|\frac{dq}{dz}\right| |dq| \\ &= \frac{1}{2\pi|q|}\int_C \rho_s^z |dq| \\ &= \frac{1}{2\pi}\int_0^{2\pi}\rho_s^z(\phi)\,d\phi. \end{aligned} \tag{32}$$

The charge density is given by (31), which is expressed as a function of ϕ in (32) by using (8), which becomes, with $q = |q|e^{j\phi}$,

$$s = P(-\ln R + j\phi) \text{ outer coax} \tag{33}$$

$$s = P(j\phi) \text{ inner coax.} \tag{34}$$

Using the fact that the contour Γ_T in (2) is twice $\Gamma_1 + \Gamma_2$, that $Q = 2Q^z$, and also that $\rho_{s_{1,2}}^z(\phi)$ is an even function of ϕ, it follows that

$$\alpha_{c_{1,2}} = \frac{R_{s_{1,2}}}{8\pi^2 Z_0}\int_0^\pi \rho_{s_{1,2}}^z(\phi)\,d\phi \tag{35}$$

where the subscripts denote different values for the two contours, with ρ_{s1}^z and ρ_{s2}^z found using (33) and (34), respectively.

The total α_c for the structure of Fig. 1, with dimension $b = b_0$ given from (25), is then given as the sum of α_{c1} and

α_{c2}. For an arbitrary b, the attenuation should be scaled by b_0/b. Hence, for a general stripline,

$$\alpha_c = \frac{\pi \operatorname{dn}(a)}{bk^2 \operatorname{sn}(a) \operatorname{cn}(a)} (\alpha_{c1} + \alpha_{c2}). \tag{36}$$

IV. Extraction of Singularity

The function $\rho_{s1}^z(\phi)$ in (35) has a singularity at $\phi = \phi_0$ corresponding to point D in Fig. 2(a). At this point, the denominator of (31) is zero, so that

$$\begin{aligned} s = s_0 &= e_1 - k^2 r^2 \\ &= e_1 + k^2(e_2 - e_1) \end{aligned} \tag{37}$$

where (17), (5), and (6) have been used. Letting $s = s_0 + \Delta s$ and expanding the denominator of (31), it may be easily shown that near s_0

$$\rho_s^z \cong \frac{\sqrt{2} A_1 (e_1 - s_0)^{1/4}}{|s - s_0|^{1/2}} \tag{38}$$

where

$$A_1 = \frac{1}{2\pi} \frac{k^2 \operatorname{sn}(a)}{r} \left(\frac{|r^2 + \operatorname{sn}^2(a)(s_0 - e_1)|}{|kr + \sqrt{e_1 - s_0}|} \right)^{1/2}. \tag{39}$$

To examine the singular behavior in the ϕ plane, the expansion

$$s \cong s_0 + \left(\frac{ds}{d\phi} \right)_{\phi_0} \Delta\phi \tag{40}$$

where $\Delta\phi = \phi - \phi_0$ is used in (38), which results in

$$\rho_s^z \cong \frac{\sqrt{2} A_1 (e_1 - s_0)^{1/4}}{\left| \dfrac{ds}{d\phi} \right|_{\phi_0} \sqrt{|\phi - \phi_0|}}. \tag{41}$$

Using (33) and (30), the derivative term in the denominator may be evaluated, resulting in

$$\rho_s^z \cong \frac{A_2}{\sqrt{|\phi - \phi_0|}} \tag{42}$$

where

$$A_2 = \frac{A_1}{|(s_0 - e_2)(s_0 - e_3)|^{1/4}}. \tag{43}$$

Although the singular behavior of (42) is integrable, this term may be extracted from ρ_{s1}^z in (35) and then analytically evaluated. The result is

$$\alpha_{c1} = \frac{R_{s1}}{8\pi^2 Z_0} \int_0^{\pi} \left[\rho_s^z(\phi) - \frac{A_2}{\sqrt{|\phi - \phi_0}} \right] + \frac{R_{s1} A_2}{8\pi^2 Z_0} \left[2\sqrt{\phi_0} + 2\sqrt{\pi - \phi_0} \right]. \tag{44}$$

In order to determine the numerical value of ϕ_0, (33) with $s = s_0$ is used along with (A2) to obtain

$$\operatorname{sn}\left[\sqrt{e_1 - e_3}(-\ln R + j\phi_0) \right] = \sqrt{\frac{e_1 - e_3}{s_0 - e_3}} = \frac{1}{\operatorname{dn}(a,k)} \tag{45}$$

where $\operatorname{sn}(z) = \operatorname{sn}(z, k_2)$ in the above expression. Using (11), (13), this may be written as

$$\operatorname{sn}(K(k_2) + jy) = \frac{1}{\operatorname{dn}(a,k)} \tag{46}$$

where

$$y = \phi_0 \sqrt{e_1 - e_3}. \tag{47}$$

Using

$$\operatorname{sn}(K(k_2) + jy) = \frac{1}{\operatorname{dn}(y, k_2')} \tag{48}$$

yields

$$\operatorname{dn}(y, k_2') = \operatorname{dn}(a,k) \tag{49}$$

from which it follows that

$$\operatorname{sn}(y, k_2') = \frac{k \operatorname{sn}(a,k)}{k_2'} = k \tag{50}$$

using (22). From Eqs. (22), (5), (7), (17), (18), the final result is

$$\phi_0 = \frac{\ln\left(\dfrac{1}{R}\right)}{K(k_2)} \operatorname{sn}^{-1}(k, k_2'). \tag{51}$$

V. Calculation Procedure and Results

The most straightforward manner of implementing the above theory is summarized in the steps below.

1) Choose a given value of Z_0. Equation (19) is then solved for $k_2 = \operatorname{cn}(a,k)$. R is then known from (20).
2) Choose a given value of k. This determines w/b and t/b from (23), (24).
3) The location and amplitude of the singularity in the ϕ integration for α_{c1} are determined from (51) and (43).
4) The attenuation constants α_{c1} and α_{c2} are calculated from (44) and (35), respectively.
5) The stripline attenuation constant α_c is found from (36).

To illustrate the above calculation, Table I shows results for the normalized attenuation $\alpha_c b/(R_s\sqrt{\epsilon_r})$ versus w/b for a stripline with a strip thickness of $t/b = 0.001$. Also shown for comparison are the results from the narrow and wide Cohn formulas, and the percent error when compared to the present method ("exact method"). Note that the optimum joining point between the two approximate formu-

TABLE I
COMPARISON OF NORMALIZED ATTENUATION CONSTANTS FOR THE CASE $t/b = 0.001$

$$\frac{\alpha_c b}{R_s\sqrt{\epsilon_r}}\quad\left(\frac{t}{b}=0.001\right)$$

w/b	Exact Method	Narrow Strip Cohn Formula	% Error	Wide Strip Cohn Formula	% Error
0.124	2.4137 E-02	2.4215 E-02	0.32	1.7145 E-02	29.0
0.175	2.0178 E-02	2.0356 E-02	0.88	1.6174 E-02	19.8
0.247	1.7087 E-02	1.7432 E-02	2.02	1.5044 E-02	12.0
0.350	1.4658 E-02	1.5304 E-02	4.40	1.3781 E-02	5.98
0.501	1.2713 E-02	1.3927 E-02	9.55	1.2425 E-02	2.27
0.604	1.1868 E-02	1.3557 E-02	14.2	1.1725 E-02	1.20
0.733	1.1081 E-02	1.3468 E-02	21.5	1.1018 E-02	0.57
0.901	1.0332 E-02	1.3802 E-02	33.6	1.0307 E-02	0.24
1.125	9.6037 E-03	1.4918 E-02	55.3	9.5933 E-03	0.11
1.438	8.8855 E-03	1.7971 E-02	102.2	8.8791 E-03	0.079
1.909	8.1705 E-03	2.9618 E-02	262.5	8.1641 E-03	0.076

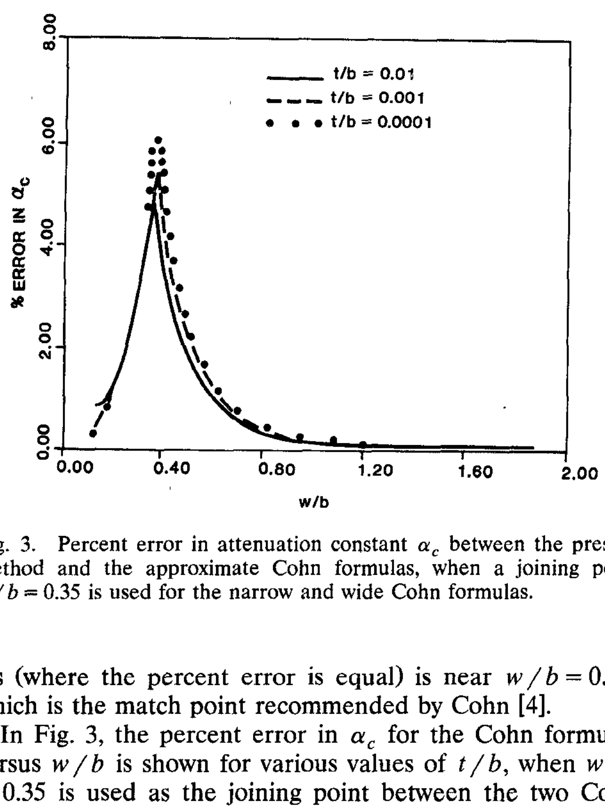

Fig. 3. Percent error in attenuation constant α_c between the present method and the approximate Cohn formulas, when a joining point $w/b = 0.35$ is used for the narrow and wide Cohn formulas.

las (where the percent error is equal) is near $w/b = 0.35$, which is the match point recommended by Cohn [4].

In Fig. 3, the percent error in α_c for the Cohn formulas versus w/b is shown for various values of t/b, when $w/b = 0.35$ is used as the joining point between the two Cohn formulas. The maximum error occurs near the joining point in all cases, and is fairly insensitive to the strip thickness. The maximum error is approximately 5–6%.

VI. CONCLUSIONS

Based on a modification of a conformal mapping originally introduced by Bates, an exact expression for the quasi-static conductive attenuation constant α_c of a general stripline having arbitrary dimensions has been obtained. The method is based on a quasi-static formula for α_c which assumes a TEM mode, and is thus restricted to the case where the skin depth is small compared to the strip thickness. The final formula for α_c involves an integral of only a smoothly varying function, since an analytic extraction of a term representing the singular behavior of the charge density was performed in the analysis.

A comparison of results between the present method and the approximate formulas of Cohn show that a maximum error of about 5–6% occurs in the Cohn formulas at the point $w/b = 0.35$ where the two approximate formulas are joined.

APPENDIX
COMPUTATION OF ELLIPTIC FUNCTIONS

A. Weierstrass Function

The Weierstrass function $P(z)$ is defined by the relation

$$z = \int_\infty^{P(z)} \frac{ds}{2\sqrt{(s-e_1)(s-e_2)(s-e_3)}} \tag{A1}$$

where e_1, e_2, e_3 are any parameters which satisfy (9). Differentiating, (30) is obtained. The Weierstrass function may be expressed as [6, p. 308]

$$P(z) = e_3 + \frac{e_1 - e_3}{\mathrm{sn}^2\left(z\sqrt{e_1 - e_3}\right)} \tag{A2}$$

where $\mathrm{sn}(w) = \mathrm{sn}(w, k_2)$ is the Jacobian elliptic function (discussed below), and k_2 is given by (15).

B. Jacobian Elliptic Functions

Denote

$$y = \mathrm{sn}(x, k) \tag{A3}$$

$$q = e^{-\pi K'/K} \tag{A4}$$

$$v = \frac{\pi x}{2K} \tag{A5}$$

where

$$K = K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}} \tag{A6}$$

is the complete elliptic integral of the first kind.

For $k \leqslant 0.0523$, the identity

$$y^2 = \frac{dc^2(x) - 1}{dc^2(x) - k^2} \quad \text{(A7)}$$

is used where $dc(x) = \mathrm{dn}(x)/\mathrm{cn}(x)$ is given by a rapidly convergent series [7, (16.23.7)]

$$dc(x) = \frac{\pi}{2K}\sec(v) + \frac{2\pi}{K}\sum_{n=0}^{\infty}\frac{(-1)^n q^{2n+1}}{1-q^{2n+1}}\cos(2n+1)v. \quad \text{(A8)}$$

For $k > 0.0523$, the identity $y = \sqrt{1 - \mathrm{cn}^2 x}$ is used, where $\mathrm{cn}(x)$ is calculated from the following rapidly converging series ([7, Eq. (16.23.2)]

$$\mathrm{cn}(x) = \frac{2\pi}{kK}\sum_{n=0}^{\infty}\frac{q^{n+1/2}}{1+q^{2n+1}}\cos(2n+1)v. \quad \text{(A9)}$$

The other Jacobian elliptic functions may be obtained from $\mathrm{sn}(x)$.

C. Jacobi Zeta Function

$Z(u)$ is defined as

$$Z(u) = \int_0^u \left[\mathrm{dn}^2(u,k) - \frac{E(k)}{K(k)}\right] du \quad \text{(A10)}$$

which may be expressed as [6]

$$Z(u) = E(u) - \frac{E(k)}{K(k)}u \quad \text{(A11)}$$

where

$$E(u) = E(k,\phi) = \int_0^{\phi}\sqrt{1 - k^2\sin^2\phi}\, d\phi \quad \text{(A12)}$$

and

$$\sin\phi = \mathrm{sn}(u,k). \quad \text{(A13)}$$

$E(k,\phi)$ is the incomplete elliptic integral of the second kind, while $K(k)$ and $E(k)$ denote the complete elliptic integrals of the first and second kind, respectively. The function $E(k,\phi)$ in (A12) may be evaluated numerically using, e.g., a Gaussian quadrature algorithm.

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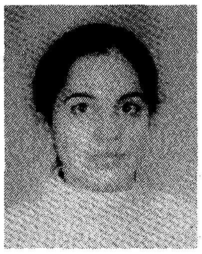

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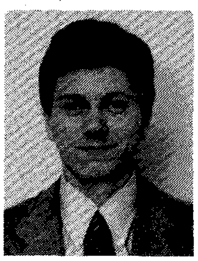

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